Space for marks	Question No.

Solution key 5.

SE - IV   AM - IV   CE.

Q1.a. $\lambda = 1, 4$.

$4A^{-1} + 3A + 2I$.

$\lambda = 1 \quad 4 + 3 + 2 = 9$

$\lambda = 4 \quad 1 + 12 + 2 = 15$.

Q1.b. $\int_0^{1+i} z^2 dz$

i) along line $y = x$.

$dy = dx$

$dz = (1+i)dx$

$$I = \int_0^{1+i} (x+iy)^2 dz = \int_0^1 (x^2 - y^2 + 2ixy)(1+i)\,dx$$

$$= \int_0^1 2i(1+i)x^2 dx = \frac{2}{3}(i-1)$$

ii) Along $y^2 = x$

$dx = 2y\,dy$.

$dz = dx + i\,dy = (2y + i)dy$.

$$I = \int_0^1 (x^2 - y^2 + 2ixy)(2y+i)\,dy.$$

$$= \int_0^1 (2y^5 - 4y^3) + i(5y^4 - y^2)\,dy.$$

$$= \left(\frac{y^6}{3} - y^4\right) + i\left(y^5 - y^3/3\right)\Big|_0^1 = \left(\frac{1}{3} - 1\right) + i(1 - 1/3)$$

$$= \frac{2}{3}(i-1).$$

Q1.c

X	578	572	570	568	572	570	570	572	596	584	
d_i	-2	-8	-10	-12	-8	-10	-10	-8	6	4	-48
d_i^2	4	64	100	144	64	100	100	64	36	16	864

$$\bar{X} = A + \Sigma d_i/n = 580 - \frac{48}{10} = 575.2$$

$$s^2 = \frac{1}{n}\Sigma d_i^2 - \left(\frac{\Sigma d_i}{n}\right)^2 = 633.6$$

$s = 68.16$

$H_0: \mu = 577$ kg $\quad H_1: \mu \neq 577$ two-tailed

$$|t| = \frac{\bar{x} - \mu}{s/\sqrt{n-1}} = \frac{575.2 - 577}{\sqrt{68.16/9}} = -.65$$

Los $= \alpha = 5\%$

Tab value of t_{10-1} dof at 5% los $= 2.25$

$\therefore$ Cal $t <$ Tab t

Accept H_0.

Q1.d H_0: There is no relation between literacy and smoking

$\chi^2 = 9.19$

Tab $\chi^2 = 3.841$

$\therefore$ Cal $\chi^2 >$ Tab χ^2

Reject H_0.

There is relation between two.

Q1.e Adding slack variable.

Max $Z = 10x_1 + x_2 + x_3 + 0s_1 + 0s_2$

St

$x_1 + x_2 - 3x_3 + s_1 = 10$

$4x_1 + x_2 + x_3 + s_2 = 20$

$-3-\frac{1}{54}$

$1-\frac{5}{2}$

	c_j		10	1	1	0	0		
CB	Basic Var B	rhs b_i	x_1	x_2	x_3	s_1	s_2	$\theta = b_i/x_j$	
	0	s_1	10	1	1	-3	1	0	$10/1 = 10$
	0	s_2	20	4	1	1	0	1	$20/4 = 5 \rightarrow$
	z_j		0	0	0	0	0	0	
	$\Delta_j = c_j - z_j$			$+10\uparrow$	1	1	0	0	
$R_1 - R_2$	0	s_1	5	0	3/4	-7/4	1	-1/4	
$R_2/4$	10	x_1	5	1	1/4	1/4	0	1/4	
	z_j			10	10/4	10/4	0	10/4	
	$\Delta_j =$			0	-6/4	-6/4	0	-10/4	

$\therefore$ all $\Delta_j \leq 0$. provide optimal sol^n

$x_1 = 5 \quad x_2 = 0 \quad x_3 = 0.$

Max Z $= 50.$

Q1.e $\partial f/\partial x_i = 0. \quad x_1 = 3, \; x_2 = 4 \quad x_3 = 5.$

SI

$$H = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Principal minors are 2, 4, 8.

$\therefore$ all are +ive, Z is min at $X_0(3,4,5)$

$Z_{min} = -50.$

Q2.a. $\lambda = (1+\lambda)^2(3-\lambda) = 0.$ ch eq^n

$\lambda = -1, -1, 3$

For $\lambda = -1$. $(A - \lambda I)X = 0.$

$$\begin{bmatrix} -8 & 4 & 4 \\ -8 & 4 & 4 \\ -16 & 8 & 8 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

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Rank $= 1$.

no. of linearly ind sol^n $= n-r = 2 = GM$

$AM = 2$

$\therefore AM = GM$ diagonalizable.

$$X = \begin{bmatrix}1\\0\\2\end{bmatrix} + t\begin{bmatrix}1\\2\\0\end{bmatrix}$$

$\lambda = 3$. $X = [1, 1, 2]'$

$$M = \begin{bmatrix}1 & 1 & 1\\0 & 2 & 1\\2 & 0 & 2\end{bmatrix} \qquad D = \begin{bmatrix}-1 & 0 & 0\\0 & -1 & 0\\0 & 0 & 3\end{bmatrix}$$

Q2.b

$$\frac{1}{z(z+1)(z-2)} = \frac{a}{z} + \frac{b}{z+1} + \frac{c}{z-2}$$

$a = -1/2$ $\quad b = 1/3$ $\quad c = 1/6$

i) $0 < |z| < 1$

$$f(z) = \frac{-1}{2z} + \frac{1}{3}(1+z)^{-1} - \frac{1}{12}(1-z/2)^{-1}$$

ii) $1 < |z| < 2$

$$\frac{-1}{2z} + \frac{1}{3z}\left(1+\frac{1}{z}\right)^{-1} - \frac{1}{12}\left(1-\frac{z}{2}\right)^{-1}$$

iii) $|z| > 2$

$$\frac{-1}{2z} + \frac{1}{3z}\left(1+\frac{1}{z}\right)^{-1} + \frac{1}{6z}\left(1-\frac{2}{z}\right)^{-1}$$

2.c.

$H_0: \mu_1 = \mu_2$

$\mu_1 \neq \mu_2$

$$s = \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2}{n_1+n_2-2}} = \sqrt{\frac{21000}{13}} = 40.19$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{110}{20.8} = 5.288$$

$\alpha = 5\%$. $\quad t_{8+7-2 \text{ dof}} = 2.16$

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$\therefore$ Cal t > Tab t.
Reject H_0. $\mu_1 \neq \mu_2$. difference is insufficient

Q2.d. $\bar{x} = \frac{\sum f_i x_i}{390} = \frac{195}{390} = 0.5$

H_0: Fit is good.
Test statistic $\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}$

E_i 242 116.7 28 $\underbrace{4.5 \quad .5 \quad .1 \quad 0}_{6}$

$\chi^2 = 40.938.$
Dof $= 7-1-1-3 = 2$ dof.
Tab $\chi^2 = 5.99.$
$\therefore$ Cal χ^2 > Tab χ^2
Reject H_0. Fit is not good.

Q2.e. Max $Z = 3x_1 - x_2 + 0S_1 - 0S_2 + 0S_3 - MA_1$
st

$2x_1 + x_2 + S_1 = 2$
$x_1 + 3x_2 - S_2 + A_1 = 3$
$0x_1 + x_2 + S_3 = 4$

C_B	Basic Var B	rhs b_i	x_1	x_2	s_1	s_2	s_3	A_1	θ
			3	-1	0	0	0	$-M$	
0	s_1	2	2	1	1	0	0	0	2
$-M$	A_1	3	1	$\boxed{3}$	0	-1	0	1	1 $\rightarrow$
0	s_3	4	0	1	0	0	1	0	4
	z_j		$-M$	$-3M$	0	M	0	$-M$	
	Δ_j		$3+M$	$-1+3M\uparrow$	0	$+M$	0	0	
0	s_1	1	5/3	0	1	1/3	0		
1	x_2	1	1/3	1	0	-1/3	0		
0	s_3	3	-1/3	0	0	1/3	1		
	z_j		1/3	1	0	-1/3	0		
	Δ_j		$2/3\uparrow$	0	0	1/3	0.		
3	x_1	3/5	1	0	3/5	1/5	0		
-1	x_2	4/5	0	1	-1/5	-2/5	0		
0	s_3	16/5	0	0	1/5	2/5	1		
			3	-1	$\cancel{9/5}$ 2	6/5	0		
	Δ_j		0	0	-2	-6/5	0.		

$x_1 = 3/5 \quad x_2 = 4/5 \quad z = 1.$

Q2.F $\quad \frac{\partial f}{\partial x_1} - \lambda_1 \frac{\partial h_1}{\partial x_1} - \lambda_2 \frac{\partial h_2}{\partial x_1} = 0 \qquad 10 - 2x_1 - \lambda_1 + \lambda_2 = 0$ (1)

$\frac{\partial f}{\partial x_1} - \lambda_1 \frac{\partial h_1}{\partial x_1} - \lambda_2 \frac{\partial h_2}{\partial x_2} = 0 \qquad 10 - 2x_2 - \lambda_1 - \lambda_2 = 0$ 2

$\lambda_1 (x_1 + x_2 - 8) = 0$ 3

$\lambda_2 (-x_1 + x_2 - 5) = 0$ 4

$x_1 + x_2 - 8 \leq 0$ 5

$-x_1 + x_2 - 5 \leq 0$ 6

$x_1 \; x_2 \geq 0$

$\lambda_1 \; \lambda_2 \geq 0$

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Case I: $\lambda_1 = 0 \quad \lambda_2 = 0.$

$10 - 2x_1 = 0 \qquad x_1 = 5$

$x_2 = 5$

Not satisfy (5) Reject

Case II $\lambda_1 = 0 \quad \lambda_2 \neq 0$

$x_2 = 7.5 \qquad x_1 = 10 - 7.5 = 2.5$

Do not satisfy 5. Reject.

Case III $\lambda_1 \neq 0 \quad \lambda_2 = 0.$

Eliminate λ_1

$-2x_1 + 2x_2 = 0 \qquad x_1 = x_2$

$\therefore x_1 = 4 \qquad x_2 = 4 \qquad \lambda_1 = 2$

Satisfy 5, 6, 7 8. $\qquad Z_{max} = 48$

Case IV From 3, 4, 5

$x_2 = 6.5 \qquad x_1 = 1.5$

$\lambda_1 = 2 \quad \lambda_2 = -5 < 0 \qquad$ Reject

Q3.a.
$$A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$$

$\lambda^3 - 5\lambda^2 + 8\lambda - 4 = 0.$

$\lambda = 2, 2, 1.$

Minimal polynomial $x^2 - 3x + 2. = A^2 - 3A + 2I$

$$A^2 = \begin{bmatrix} 13 & -18 & -18 \\ -3 & 10 & 6 \\ 9 & -18 & -14 \end{bmatrix}$$

$\therefore$ Degree of minimal poly < Order of A.

Derogatory.

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Q3.b Pole $(z-1)^2(z+1)=0$.

$z=1$ pole of order 2

$z=-1$ simple pole.

i) $C=|z|=1/2$

both pole lie outside C.

$$\int_C f(z)\,dz = 0$$

ii) $C=|z|=2$

Both pole lie inside C.

$$\text{Resi (at } z=-1) = \lim_{z\to -1} (z+1)\frac{z^2}{(z+1)(z-1)^2}$$

$$= \lim_{z\to -1} \frac{z^2}{(z-1)^2} = 1/4$$

$$\text{Resi (at } z=1) = \lim_{z\to 1} \frac{d}{dz}(z-1)^2\frac{z^2}{(z-1)^2(z+1)}$$

$$= \lim_{z\to 1} \frac{z^2+2z}{(z+1)^2} = \frac{3}{2}$$

Resi Th

$$\int f(z)\,dz = 2\pi i\left(\frac{1}{4}+\frac{3}{2}\right) = \frac{7\pi i}{2}$$

Q3.c $\frac{e^{-\lambda}\lambda}{1} = 2\frac{e^{-\lambda}\lambda^2}{2!}$

$\lambda=1$.

$P(X=3) = \frac{e^{-1}\,1^3}{3!} = 0.0613.$

Q3.d. Σx_i 100 95 135 $= 330$ T

$N=15$

Correction factor $= C = T^2/N = 7260$

$SST = \Sigma\Sigma x_{ij}^2 = 7590$

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$$\therefore SST = \sum\sum x_{ij}^2 - \frac{T^2}{N} = 7590 - 7260 = 330$$

$$SSC = \frac{(\sum x_1)^2 + (\sum x_2)^2 + (\sum x_3)^2}{n_1} - \frac{T^2}{N}$$

$$= \frac{(100)^2 + (95)^2 + (135)^2}{5} - 7260 = 190$$

$$SSE = 140. = SST - SSC$$

Sources of variation	Sum of squares	dof	Mean sum of sq	F
Between sample	SSC = 190	2	190/2 = 95	$F = \frac{MSC}{MSE}$
Within sample	SSE 140	12	140/12 = 11.67	= 8.14
Total	SST 330	14		

$$\text{Tab } F_{2,12 \text{ dof at } 5\%} = 3.88$$

$\therefore$ Cal F > Tab F   Reject H_0.

Q3.de   Max $z^* = -z = -2x_1 - 2x_2 - 4x_3$

$$-2x_1 - 3x_2 - 5x_3 + s_1 = -2$$
$$3x_1 + x_2 + 7x_3 + s_2 = 3$$
$$x_1 + 4x_2 + 6x_3 + s_3 = 5$$

			-2	-2	-4	0	0	0
C_B	B	b_i	x_1	x_2	x_3	s_1	s_2	s_3
0	s_1	-2	-2	-3	-5	1	0	0
0	s_2	3	3	1	7	0	1	0
0	s_3	5	1	4	6	0	0	1
z_j			0	0	0	0	0	0
$\Delta_j = c_j - z_j$			-2	$-2 \uparrow$	-4	0	0	0
-2	x_2	$2/3$	$2/3$	1	$5/3$	$-1/3$	0	0
0	s_2	$7/3$	$7/3$	0	$16/3$	$1/3$	1	0
0	s_3	$-5/3$	$-5/3$	0	$-2/3$	$4/3$	0	1
Δ_j			$-4/3$	0	$-2/3$	$-2/3$	0	0

Inc... ve...

$\Delta_j = \frac{\Delta_j}{a_{ij}}$

$x_1 = 0 \quad x_2 = 2/3$

$Z_{min} = 4/3.$

Q3. R $\quad L(x_1, x_2, \lambda) = 4x_1 + 8x_2 - x_1^2 - x_2^2 - \lambda(x_1 + x_2 - 4)$

$\frac{\partial L}{\partial x_i} = 0 \qquad \frac{\partial L}{\partial \lambda} = 0.$

$\frac{\partial L}{\partial x_1} = 0 = 4 - 2x_1 - \lambda = 0.$

$\frac{\partial L}{\partial x_2} = 0 \quad 8 - 2x_2 - \lambda = 0$

$\frac{\partial \lambda}{\partial L} = -(x_1 + x_2 - 4)$

$\lambda = 2, \quad x_1 = 1$

$$\Delta = \begin{vmatrix} 0 & 1 & 1 \\ 1 & -2 & 0 \\ 1 & 0 & -2 \end{vmatrix} = 4$$

$\therefore \Delta$ is +ive $\quad X_0$ is maxima

$Z_{max} = 18.$